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Short Communication

Oscillation hysteresis in a two-mass model of the vocal folds

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1. Introduction

Using the qualitative theory of dynamical systems, the onset and offset of the vocal fold oscillation at phonation has been described by an oscillation hysteresis model [1,2]. This model is built from the combination of a cyclic fold bifurcation for limit cycles, where a stable and an unstable limit cycle are generated, with a subcritical Hopf bifurcation, where the unstable limit cycle is absorbed. The former bifurcation corresponds to oscillation offset, and the latter to oscillation onset. Thus, onset and offset occur at different bifurcations, and consequently at different values of the control parameter. This model would explain experimental results which show that voice onset and offset during speech occur at different biomechanical configurations of the phonatory system. For example, the subglottal pressure is higher at oscillation onset than at oscillation offset [3], the intraoral pressure is lower [4], the airflow is lower [5], and the glottal width is smaller [6].

In previous works [1,2], the existence of a subcritical Hopf bifurcation was shown by applying the Hopf Bifurcation Theorem [7] to a simple mucosal wave model of the vocal folds [8]. Although the simplicity of this model permitted the theoretical treatment in qualitative terms, it does not allow realistic simulations of the oscillation hysteresis.

It would be interesting, as next step, to study the oscillation hysteresis using more elaborated models of the vocal folds. Further, it is desirable to study this phenomenon in terms of voice output parameters such as intensity and fundamental frequency. Such output parameters are

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easier to measure experimentally, and thus the analytical results could be better compared with experimental data. In this note, a two-mass model of the vocal folds [9] coupled to a two tube approximation of the vocal tract [10] will be used for such study.

2. Model

In the two-mass model [9], each vocal fold is represented by two mass-damper–spring oscillators coupled through a spring. Fig. 1 shows a sketch of the model. The left and right vocal folds are assumed identical, and they move symmetrically with respect to the glottal midline, in the horizontal direction. The glottal aerodynamics is described following an approximation of the boundary layer model [11] for high Reynolds numbers. The vocal fold model is coupled to a two-tube approximation of the vocal tract, with dimensions corresponding to a male vocal tract for vowel /a/ (Fig. 2): $S_1 = 1\,\text{cm}^2$, $S_2 = 7\,\text{cm}^2$, $L_1 = 9\,\text{cm}$, $L_2 = 8\,\text{cm}$. Its equations are derived using a transmission line analogy, terminated in a radiation

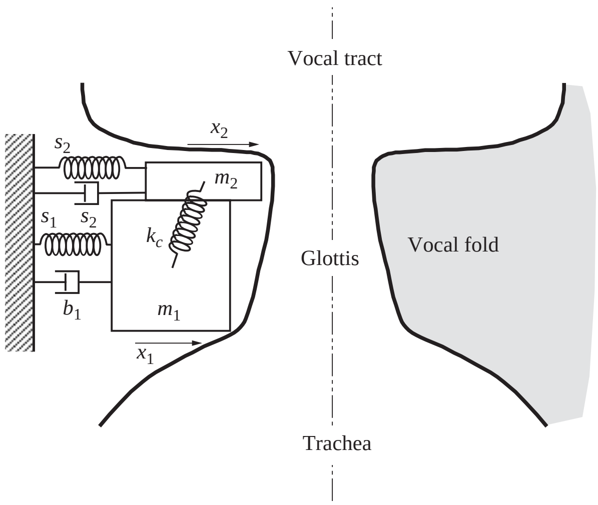

Fig. 1. Two-mass model of the vocal folds.

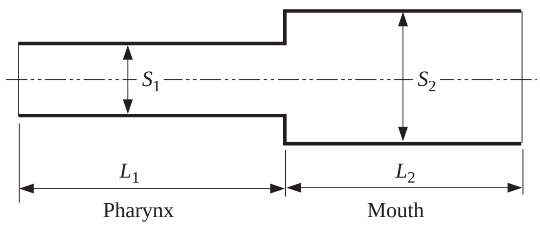

Fig. 2. Two-tube approximation of the vocal tract for vowel /a/.

load of a circular piston in an infinite baffle [9,10]. Both the two-mass and the vocal tract models are well-known models widely used for voice simulations, and their complete equations may be found in the indicated references.

Voice intensity is computed as $\mathrm{SPL} = 115 + 10\log_{10}\bar{P}$, where SPL is the sound pressure level (in dB) and $\bar{P}$ is the time-averaged acoustic power (in watts) produced by the model [10].

3. Onset–offset bifurcations

Let us review briefly the phenomenon of oscillation hysteresis. According to the theory of dynamical systems, the qualitative change of dynamical behavior at a critical value of a parameter is called a bifurcation. Fig. 3 shows a bifurcation diagram for oscillation hysteresis. There are two bifurcations: a Hopf bifurcation, at which an equilibrium position changes stability and an unstable limit cycle is generated, and a cyclic fold bifurcation at which the unstable limit cycle and a stable second limit cycle coalesce and cancel each other.

Suppose that the control parameter is increased from zero. At the Hopf bifurcation, the oscillation will start and will increase its amplitude rapidly to the stable limit cycle. If the parameter is now decreased, the oscillation will follow the curve corresponding to the stable limit cycle, until reaching the cyclic fold bifurcation. At this point, it will vanish abruptly. Thus, oscillation onset and offset occur at different threshold values of the parameter with a hysteresis effect. Note that between the onset and offset thresholds, two stable states co-exist: an equilibrium position and a stable limit cycle.

In the present case, voice onset–offset bifurcations were determined by solving numerically the equations of the model. Bifurcations in terms of the voice intensity and fundamental frequency were sought. Thus, the subglottal pressure P_s was used as main control over voice intensity.

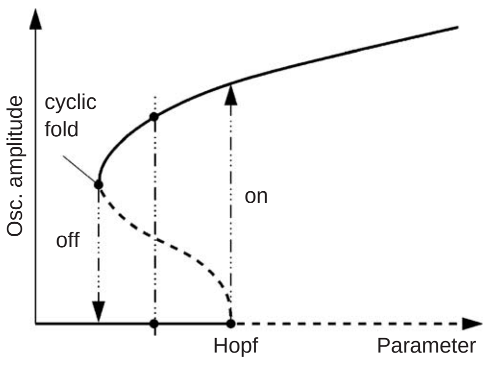

Fig. 3. Oscillation hysteresis phenomenon. The curves in full and broken lines represent a stable and an unstable limit cycle, respectively. Along the horizontal axis, the full and broken line regions represent the stable and unstable regions of an equilibrium position. Oscillation onset occurs at a subcritical Hopf bifurcation, and offset occur at a cyclic fold bifurcation.

Fundamental frequency was controlled through a scaling parameter Q for the natural frequencies of the two-mass model [9].

Voice onset and offset were obtained by increasing the subglottal pressure from zero until obtaining a voice output, and next decreasing it to zero (see Fig. 4). The rate of variation of subglottal pressure was taken proportional to the oscillation frequency, at 10 Pa/cycle. Voice onset was then defined as the time at which the oscillation amplitude starts to increase, or equivalently, the time at which its first derivative becomes greater than zero. The definition of voice offset was based on the observation that at oscillation offset, a sudden decrease of the oscillation amplitude occurred. Voice offset was then defined as the time of this sudden decrease, or equivalently, as the time at which the second derivative of the oscillation amplitude is zero. This definition was verified by observing that the oscillation takes a sustained amplitude if the decrease of subglottal pressure is interrupted before that point, whereas it vanishes beyond that

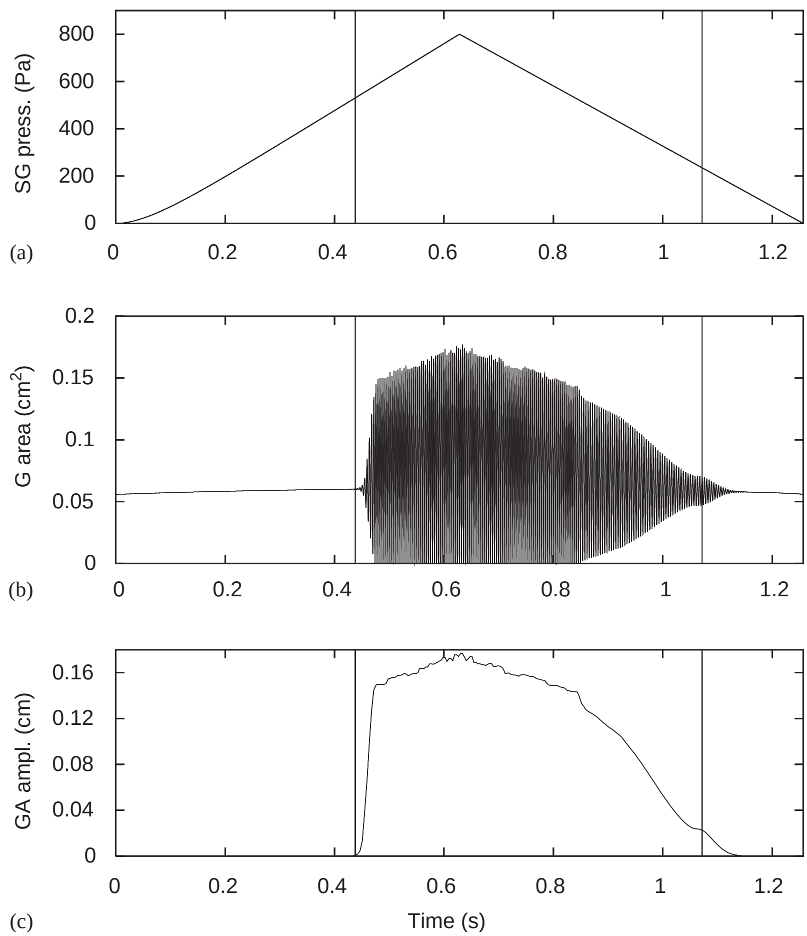

Fig. 4. Simulation of glottal area when varying the subglottal pressure: (a) subglottal pressure; (b) glottal area; (c) oscillation amplitude of glottal area.

point. Fig. 4 shows that voice onset and offset occur at different values of the subglottal pressure, in a clear hysteresis phenomenon.

The rate of variation of the subglottal pressure affects the onset–offset hysteresis. Fig. 5 shows the case of a smooth up and down variation of the subglottal pressure from zero, to a maximum value smaller the onset threshold but higher than the offset threshold. In this case, no oscillation results. Fig. 6 shows the case of an abrupt variation of the subglottal pressure between the same limits as above. In this case, an oscillation appears. Recall that between the offset and onset thresholds, two stable states coexist. Although the oscillation onset threshold (that is, the Hopf bifurcation value) is not reached, the sudden change of subglottal pressure kicks the system out of the basin of attraction of the equilibrium position and into the basin of the stable limit cycle.

The bifurcation diagram in Fig. 7 shows the voice onset–offset thresholds on subglottal pressure versus fundamental frequency. The onset threshold is always higher that the offset threshold, and both increase almost linearly with frequency. The ratio between them is 0.45 to 0.72, matching experimental measurements [3] and the theoretical range of 0.5–1 obtained on the mucosal wave model [1,2].

Finally, the bifurcation diagram in Fig. 8 shows voice intensity at onset and offset versus fundamental frequency. Voice intensity at onset is higher than offset, showing again a hysteresis phenomenon. This diagram presents the onset–offset bifurcations in terms of parameters which are easy to measure experimentally, and have been traditionally used to evaluate voices in voice range profiles.

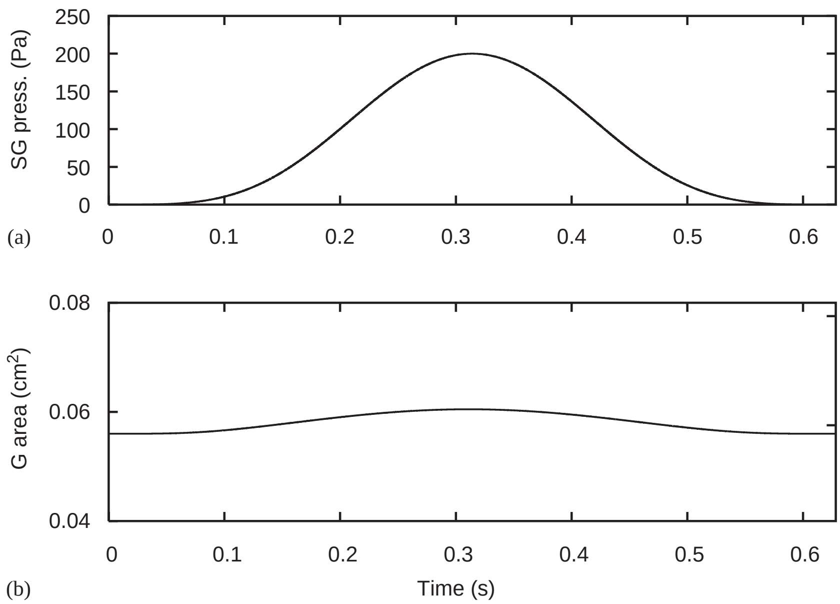

Fig. 5. Simulation of glottal area in case of a smooth variation of subglottal pressure: (a) subglottal pressure; (b) glottal area.

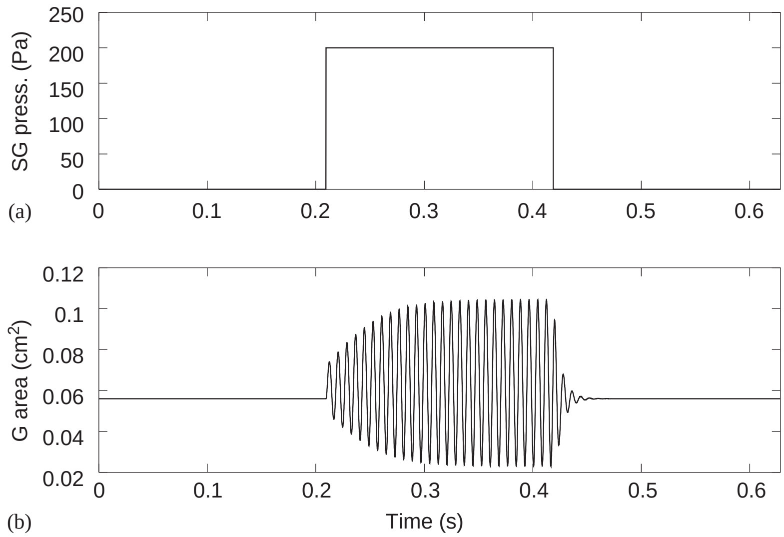

Fig. 6. Simulation of glottal area in case of an abrupt change of subglottal pressure: (a) subglottal pressure; (b) glottal area.

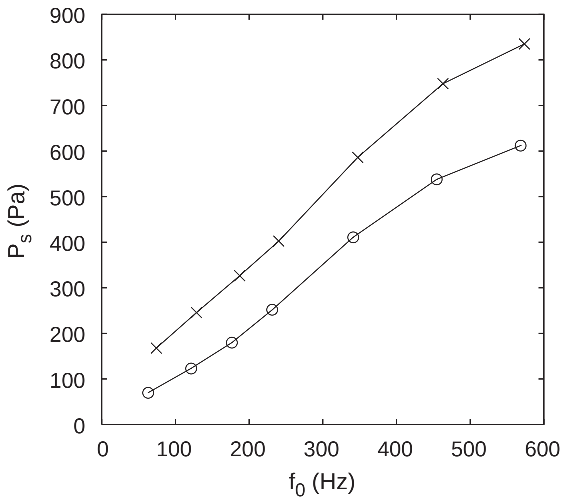

Fig. 7. Subglottal pressure at onset (crosses) and offset (circles) versus fundamental frequency.

4. Conclusions

This study has shown that voice onset–offset hysteresis may be simulated with standard mathematical models of the phonatory system. The results support the oscillation hysteresis model as a valid mathematical representation for the voice onset–offset hysteresis. They also show the possibility of studying the hysteresis in terms of voice output parameters as fundamental

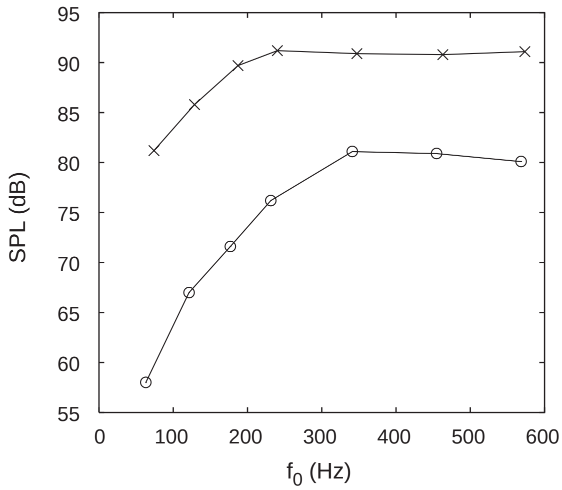

Fig. 8. Voice intensity at onset (crosses) and offset (circles) versus fundamental frequency.

frequency and voice intensity, making thus use of tools and methods common in clinical applications.

Acknowledgements

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